Public Infrastructure Underinvestment:
The Risk to Canada’s Economic Growth
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Preface
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Preface

For some time, experts have been warning of a deficit between the current state of Canadian infrastructure and what is required—the so-called “infrastructure gap.”

And, while it is well known that the quality and quantity of infrastructure has a direct impact upon how efficiently societies are able to operate and grow, individuals and businesses have yet to connect underinvestment in infrastructure to their personal prosperity.

Citing a municipal infrastructure deficit in the billions of dollars sounds staggering, but it may not resonate on a personal level with the public. In general, when large prospective numbers are used, we are at risk of becoming numb to the real meaning behind these numbers, and the real risks of a large infrastructure deficit can feel abstract.

Our findings create a clear linkage between sustained investment in infrastructure and the prosperity of individual Canadians.

Over the next 50 years there is the risk of public infrastructure underinvestment that could cost the Canadian economy 1.1 per cent of real gross domestic product (GDP) growth. The effect of this underinvestment on the Canadian public breaks down as follows: It will cost the average Canadian worker between $9,000 and $51,000, with the younger generation disproportionately at risk, and decrease the after-tax profitability of Canadian businesses by a long term average of 20 per cent. Businesses with long-term goals are at greatest risk.

These risks have largely escaped public attention, despite the recent global focus on infrastructure investment as a short-term stimulus measure.

The solution is more complex than simply investing in narrowing the infrastructure gap. That approach will not guarantee the best possible growth for the economy. It is arguably more appropriate to extend the focus to long-term infrastructure policies that maximize economic growth in the future, starting with the current state of infrastructure today. Stable infrastructure spending—as opposed to the current volatility in spending—promotes stronger economic growth and guarantees more funding will be available for infrastructure.
Long-term, mitigating the underinvestment risk would require an increase in infrastructure spending of up to 62 per cent (44 per cent increase required for new investment and 179 per cent increase for maintenance). Results show that for every extra dollar paid in taxation revenue, the taxpayer is better off by $1.48 on average, in after-tax wage terms. That means mitigating the underinvestment risk is cost effective.

This examination of the impacts of long-term infrastructure investment patterns on future economic production, income of employees, and net profits after-tax of employers, is meant to open Canadian employees’ and employers’ eyes to the critical relationship between long-term public infrastructure investment policy and their future potential economic prosperity, as well as the present-day risk they bear.

Quantifying the relationship between public infrastructure and economic prosperity is a challenging and complex task—there is limited information on these points. The research relies on mathematical modelling of demographics, labour force, production, taxation and wages to estimate the long-term connections between public infrastructure investment, both on new infrastructure and on maintenance, and economic production, real after-tax wages and real net profits after-tax. The results are not intended to dictate public infrastructure investment policy in Canada. Rather, the results emphasise the potential economic risk that Canadian employees and employers bear when long-term infrastructure trends tend towards persistent underinvestment.

**Results at a Glance**

**Degree of underinvestment:** Total public infrastructure investment over the last decade has averaged around 3.1 per cent of GDP. If adopted as a future policy, current long-term public infrastructure investment trends would be lower than what our analysis recommends. The analysis suggests that public investment in infrastructure should be increased by 62 per cent (44 per cent increase for new investment and 179 per cent increase for maintenance). Such a correction would maximize overall potential economic returns and allow the Canadian economy to achieve its potential for a greater and more stable economy.

**Societal risk of lost opportunity:** If underinvestment continues according to the last decade’s trend, it is estimated that the Canadian economy will lose the opportunity for an annual average increase of 1.1 per cent of real GDP growth over the next 50 years. A 3D diagram illuminates the risk by showing the relationship between total public infrastructure investment as a percentage of GDP, maintenance investment and the 2060 real value of GDP as a percent of 2009 GDP (vertical axis).
The 3D visualization of the results clearly illustrates that an adequately funded, stable, long-term infrastructure policy secures the foundation of long-term economic production (the plateau). It is evident that the adoption of the recent trend of about 3.1 per cent of GDP for the next 50 years puts this foundation on a risky slope in a place where reductions in infrastructure investment or misallocations could have a disproportionate impact upon the deterioration of economic growth.
**Personal risk of lost opportunity:** Employees and their employers have a stake in ensuring an adequately-funded, stable, long-term public infrastructure policy. If the potential identified risk of underinvestment were to occur, it is estimated that employees would lose the opportunity of a 0.5 per cent per annum average increase of their real after-tax income over the next 50 years. In present value terms, this represents an average lost opportunity after tax income of $18,000.

On a more personal note, this lost opportunity is greater for younger employees where the after-tax income impact ranges between $47,000 and $51,000 on average (in present value terms). That is, long-term public infrastructure underinvestment is not only individually significant, but it will disproportionately affect the younger members of the Canadian labour force. This is the effect of using infrastructure now without adequate compensation for its replacement, maintenance and growth.

**Business risk of lost opportunity:** Employers are also not immune to the risks of long-term underinvestment in public infrastructure. If the potential risk of underinvestment identified were to occur, it is estimated that Canadian employers could forgo, on average, a 0.7 per cent per annum increase of their real net profit after-tax (NPAT). This impact accumulates over time, with businesses that have long-term ambitions in Canada being disproportionately affected. For example, over the next ten years, the average real NPAT opportunity lost for employers is estimated at three per cent per year. Over the next fifty years, the average real NPAT opportunity lost for employers is estimated at 20 per cent per year.

**Infrastructure investment can be cost effective:** Funding for a potential increase in infrastructure investment naturally needs to be taken into account. The analysis assumed funding from Canadian taxpayers. The average result for the eradication of the potential identified risk of underinvestment was a net benefit for taxpayers where for every extra dollar paid in taxation revenue, an average person is better off in after-tax wage terms by $1.48.

**Stable investment is better than volatile spending:** The 62 per cent increase in infrastructure investment assumes a stable future infrastructure investment pattern. Historically, such stable investment has not been observed. Analysis shows that continued historical instability of investment has a double negative effect. Not only does irregular investment reduce the economic growth potential from infrastructure investment, but it also results in less funding being available to support infrastructure. In other words, stable infrastructure investment results in greater growth with lower overall outlays.
Conclusions

With an average annual growth of 1.1 per cent of GDP at risk, the identified risk of long-term underinvestment in public infrastructure is significant to employees, employers and society as a whole. The consequences are disproportionately placed upon the shoulders of younger members of the labour force and upon employers that have long-term ambitions in Canada. To mitigate the risk, total infrastructure spending can be increased by up to 62 per cent which includes increases to both maintenance and new investments. When coupled with recent estimates of a significant infrastructure deficit in Canada, the identified risk of infrastructure underinvestment is persuasive and in need of attention.

Further analysis had identified that unstable or short-term infrastructure policies introduce additional frictions to the economic productive process. The result is that volatile infrastructure investment does not translate as well into economic growth. In order to ensure stable funding, long-term infrastructure investment plans that take into account the current inventory of infrastructure and future demographic and economic needs are required. Such plans require a national asset management program to identify investment and maintenance priorities well in advance.

The macroeconomic analysis in this report considers only the relationship between infrastructure investment and key macroeconomic variables. It does not consider the details of infrastructure inventory currently on hand, or how such an inventory should change over time. The analysis, therefore, only considers whether the current long-term investment trend makes sense from a macroeconomic point of view.

Given the identified risk of infrastructure underinvestment, it is strongly recommended that further analysis be conducted that couples national infrastructure inventory and deficit estimates with macroeconomic policy analysis. This unification of data and analysis is necessary to ascertain an actionable strategy that can mitigate the identified risk of infrastructure underinvestment in Canada.
1.0 Introduction

Public infrastructure is the lifeline of society. Without roads, water systems, and other infrastructure, modern economies would cease to function in the way to which we have become accustomed. The quality and quantity of infrastructure has a direct impact upon how efficiently societies are able to operate and grow. The Canadian public implicitly recognizes the crucial nature of infrastructure. A recent survey by the Federation of Canadian Municipalities (FCM) indicates that 96 per cent of Canadians would like government to maintain or increase infrastructure investment (1). Infrastructure ranked second only to health care in terms of items to protect from investment cuts. This research study advances the discussion by asking, “What is the appropriate level of infrastructure investment to maximize long-term economic prosperity, and what is the impact of infrastructure investment policies upon employees and employers?”

To date, there have primarily been two types of infrastructure research completed in Canada. The first type relates to cataloguing and estimating upgrade and maintenance costs of existing infrastructure in Canada. In order to formulate any detailed long-term infrastructure strategies, knowledge of the current state of Canadian infrastructure is required. A well-publicized study was completed by Saeed Mirza in 2007 (2) who estimated the current municipal infrastructure deficit to be well over $123 billion and growing. While this first type of infrastructure research provides general information on the nature and significance of the problem, the figures are often based on controversial estimates typically prepared by engineering consultants. These estimates are subject to considerations of the useful life of infrastructure, which is not only difficult to predict but also contingent on ongoing lifecycle maintenance. Further complicating the issue is that innovative approaches, or new materials to deal with aging infrastructure repair, might also have an impact on infrastructure asset management costs in the future.

The second type of research relates to investigating how public infrastructure affects economic growth. This connection is relatively new. Initial studies investigating the interaction only began surfacing as recently as the late 1980s. Early studies by Aschauer focused on the United States (3) and other G7 members (4). Aschauer concluded that the decline in productivity growth for many of the G7 nations could at least partially be explained by declining infrastructure investment to GDP ratios. More recent studies have found that production growth correlates well with the growth of public infrastructure, with private capital growth slightly delayed (5). Other studies have indicated that infrastructure plays a particularly important role in supporting GDP growth in Canada due to its relatively low population density and more extreme
climate (6; 7). Recent support for the connection between infrastructure investment and production growth can be found in a report from the Conference Board of Canada (8) which concluded that for every real dollar spent on public infrastructure in Ontario, $1.11 in real GDP was generated.

Additional evidence to indicate that public infrastructure may influence macroeconomic dynamics can be seen by comparing the productivity in Canada to that of the United States. Given the tight integration between the U.S. and Canadian economies, it is somewhat surprising that manufacturing productivity has diverged significantly over the past decades (9). One possible explanation for at least part of the larger U.S. growth rate is more infrastructure investment. Brox (9) points out that, as the productivity gap between the United States and Canada has widened since 1994, there has been a 3.5 per cent decline in the real value of Canadian infrastructure stock, while in the United States, public infrastructure has grown by 24 per cent.

While all of these studies emphasize the importance of infrastructure to the Canadian economy as a whole, they struggle to make the connection between the importance of sufficient infrastructure and the future potential cost impact that infrastructure quality could have on different members of the stakeholder community. This study takes the second approach to another level by addressing this question through econometric modelling.

While there is a general understanding that an infrastructure gap has a number of negative consequences and a smaller one would be better, there is little work to translate the importance of sufficient infrastructure into terms that would allow the public, industry, and government to understand the present-day risk such a deficit represents. For example, a large current infrastructure deficit along with a lack of long-term planning would likely result in higher taxes on the public, reduced corporate profits, and strained government cash flows.

This study aims to emphasize the importance of long-term public capital management in terms with which stakeholders can readily identify, leading to an understanding of the risks associated with infrastructure policies. For long-term prosperity, there is a need for an appropriate balance between investment in new infrastructure and the maintenance of existing infrastructure. Following this line of inquiry, the study considers the sensitivity of future wages, corporate profits, and GDP to total infrastructure investment and this balance between new investment and maintenance.

The result is not intended to be a definitive statement as to what public infrastructure investment policy should be in Canada. Rather, the results endeavour to emphasise the potential economic risk that Canadian employees and employers bear when long-term infrastructure planning is in jeopardy.
1.1 The Canadian Infrastructure Deficit

In general, an infrastructure deficit is the amount of investment required to repair and maintain existing public infrastructure. This includes the immediate funding for required upgrades, and the future investment needed to maintain a minimal level of service. It does not include investment required to accommodate future growth. For example, a highway may require immediate repairs and annual maintenance investment until the end of its expected service life, which would be included as part of the infrastructure deficit. But the expected cost to add a new lane in the future to accommodate increased traffic due to a larger population is not.

A report commissioned by the FCM in 2007 estimated the infrastructure deficit to be $123 billion (2). This is significantly larger than the $57 billion estimate from a 2003 study led by the Canadian Society for Civil Engineering (10). The FCM report concludes with a single recommendation: that a national infrastructure plan be established, cataloguing the existing Canadian infrastructure deficit across all levels of government, that can be used as a basis for efficient infrastructure management in the future.

While a direct analysis of the inventory of infrastructure indicates a substantial deficit, there are other signs that the balance of public capital (infrastructure) and private capital are out of balance. One indicator is the rate of return on public capital. While it is difficult to accurately determine the rate of return, estimates for Canada have ranged from a low of 11 per cent to a high of 25 per cent, with several other studies falling within this range (9; 11; 12; 13). In contrast, the return on private capital is around 10 per cent to 13 per cent (11; 13). In an economy with an appropriate balance of private and public capital, one would expect that the return on private investment would be greater than the return on public investments. This implies a shortage of Canadian public infrastructure (9).

One factor contributing to the rising infrastructure deficit is a reduction in the total level of investment for infrastructure projects. In the late 1990s, annual investment in public infrastructure, as measured as a fraction of GDP, had fallen to less than half of the peak rate in the 1960s. Figure 1 highlights the steady decline in total infrastructure investment from all levels of government from 1961 to around 2004. In recent years, the trend has reversed with infrastructure investment rising slightly, but still remaining well below the peak levels set in the 1960s.
The second contribution to the rising infrastructure deficit is the decreased level of maintenance on existing infrastructure. If infrastructure is not maintained, it will decay faster than properly maintained capital, resulting in a less effective contribution to production. Figure 2 shows the recent decline in maintenance levels from the early 1990s onwards. The longer maintenance is delayed, the greater the cost required to restore the infrastructure to its full productive value.

It should be noted that the definition of what constitutes public infrastructure is not without controversy. As the focus is on a macroeconomic analysis, and in order to be consistent with other Canadian macroeconomic studies such as those by Macdonald (13), the definition of public infrastructure adopted for this project corresponds to the Statistics Canada definitions where capital and investment is decomposed in business sector and public sector contributions.

Figure 1: Total infrastructure spending as a fraction of GDP from 1961 to 2008.

Figure 2: Fraction of spending spent on repair and maintenance compared to total infrastructure spending.
The presence of an infrastructure deficit means that the Canadian economy is likely not operating as efficiently and as productively as it could be. Through long-term low maintenance investment and relatively lower new capital investments, the deficit has grown considerably in the past couple of decades (2).

In order to reduce the infrastructure deficit, forward-looking, long-term solutions are required [see (14)]. Simply reducing the infrastructure deficit does not guarantee the best possible growth for the economy. In particular, infrastructure deficit estimates do not include the cost and maintenance of new infrastructure required for future demographic and economic expansion. Therefore, rather than focussing on elimination of the infrastructure deficit, an arguably more pertinent focus is one that investigates the long-term infrastructure policies that maximize economic growth in the future, starting with the current state of infrastructure today. This argument has been made several times in ReNew Canada, the infrastructure renewal magazine, including in “Bridging the Gap” (September/October 2008).

There are several aspects to long-term planning. Successful infrastructure management relies upon both a commitment to investment, and a full lifecycle analysis of the costs involved (i.e. 15 and documents therein). However, at the highest level, there are two main factors governing the growth of infrastructure:

- The total amount spent on infrastructure
- The fraction of investment directed towards the maintenance of existing infrastructure as opposed to new capital investments

The historical values for these two parameters are shown in Figure 1 and Figure 2. Of particular interest is the outcome of following the recent infrastructure policy trend where total infrastructure investment over the last decade has averaged around 3.1 per cent, and maintenance around 15 per cent of total investment compared to other possible infrastructure policies.
1.3 Reframing the Infrastructure Investment Debate: Resonating with Canadians

Citing a municipal infrastructure deficit of $123 billion sounds staggering but might not resonate on a personal level with community members. In general, when large prospective numbers are used, we are at risk of becoming cognitively numb, with diminishing psychological sensitivity as numbers become increasingly large and more abstract (16). The effect is particularly pronounced when it is not framed in a context with which we are familiar (17). For example, simply stating that poor infrastructure management would reduce real GDP by $2.2 trillion by 2060 may elicit very little reaction from an individual worker. Alternatively, advising employees that it might cost each of them the equivalent of $50,000 today (as an example) may result in a much stronger connection with the issue. In most situations, people are risk adverse (18), and if they feel personally at risk due to infrastructure decisions, they are more likely to become involved in the conversation. Therefore, it is important to relate the macroeconomic conclusions to the individual stakeholders in a more meaningful way.

The primary stakeholders of infrastructure in Canada are individual employees, employers (for example, corporations), and society overall. To make the impact of infrastructure policy personal to each of these stakeholders, the primary outcomes of the model are after-tax wages, net profit after tax, and total GDP. The results of this analysis are put into terms that Canadian public, employees, and employers can relate to.

In Section 3, we will demonstrate how a macroeconomic analysis can be translated directly into terms to which these primary stakeholders can relate.

2.0 A 50-Year Approach to Modelling Infrastructure Policies

In order to capture and analyse the economic impacts of infrastructure policy for all stakeholders, a model must include the direct effects of infrastructure upon production and the extent to which the labour force, government revenues, and corporate profits are affected. In addition, since infrastructure management is a long-term endeavour, the study investigates the effects of infrastructure policies on the Canadian economy over the next 50 years until 2060. An overview of the model is provided in the following section. A detailed technical methodology is provided in Appendix C.
2.1 Overview of Modelling Infrastructure Policy Impacts in Canada

There are two key model components required to study the long-term effects of infrastructure policy in Canada. Over the 50 year timeframe of this study, the population in Canada is expected to change significantly. Therefore, the first component required is a demographic model to project the population from the present day until 2060. The demographic model considers four primary processes: birth, death, migration and aging. Historically observed trends, such as decreasing mortality rates, are preserved in the model.

The second key component is the economic model. It is coupled to the demographic component through detailed modelling of the labour force. The basis of the economic production model is a Cobb-Douglas (19) function which relates the total economic production \( Y \) to the labour capital \( L \), private capital \( K \) and public capital \( G \) stocks.

\[
Y = \alpha K^{\beta_k} L^{\beta_L} G^{\beta_G}
\]

The elasticities of production, \( \beta_{(K,L,G)} \), used in this study are from Macdonald (13). The elasticities of production for private and labour capital used were \( \beta_k = 0.31 \) and \( \beta_L = 0.69 \) respectively. The elasticity of production for public capital used was \( \beta_G = 0.1 \). The multifactor productivity term, \( \alpha \), accounts for changes in productivity due to other contributions such as improvements in technology. The stock of public infrastructure is governed by the two infrastructure policy parameters shown in Figure 1 and Figure 2: the total investment on infrastructure as a percentage of GDP, and the fraction of total investment that is allocated to maintenance. The effect of maintenance upon the value of the infrastructure capital stock is captured through adjustments to the depreciation rate. Higher levels of maintenance reduce the depreciation rate of infrastructure towards a lower, non-zero limit. Since public infrastructure is used by all industries in the economy, there is also an effect whereby increased production results in an increased depreciation rate due to greater wear and tear. This is the approach used in other studies to capture the importance of maintenance in infrastructure policy decisions (20). Since private capital is not shared across industries, its evolution is governed simply through new private investment, and a depreciation rate based on historical estimates.

Labour capital is derived from real average wages (by age and sex) and the employment rate in the population. As the economy grows, new employees are hired depending on availability, which is constrained by the demographic model. Real wages are adjusted according to the demand for employment with higher demand resulting in larger increases in real wages. Employee hiring and retirements are modelled based on historical trends in the labour force.

Infrastructure policy affects government expenses directly through funding of infrastructure projects, and also indirectly through tax revenues. A more productive economy will generate more taxable profits and income requiring lower tax rates for the same dollar value of infrastructure investment. In contrast, a higher level
of infrastructure investment at the same size of economy requires higher tax rates. Governments must be able to fund government salaries, infrastructure investment, and other services. If insufficient revenue is available, tax rates are adjusted to work towards eliminating the deficit. In contrast, if a surplus is present, the government will adjust tax rates to reduce the surplus.

Based on studies which correlate production growth to public infrastructure investment, the target growth in production is set to match the growth rate of public infrastructure. However, the actual growth that can be achieved is constrained by the rate at which the labour force can change (which is coupled to the demographic model), and the ability of private industry to invest in private capital.

The net result is a complex, highly coupled set of differential equations that connects many different aspects of the overall economy. For example, if production rises, the government will receive more taxation revenue, and may reduce taxes in response if it has a surplus. In addition, since overall revenue is higher, more government funding is available for infrastructure investment which may further increase production. However, if an increase in production requires more employees, the number of employees is limited by the demographic population. A shortage of employees may eventually feed back to government tax rates through limitations in production, which reduces government revenue.

The system of differential equations is solved using an adaptive time-step Runge-Kutta integrator and uses actual historical data for the initial conditions. The entire model is connected to an optimization framework which allows one to determine the policy parameters which maximize (or minimize) a given model output. For example, the infrastructure policy parameters which maximize the real GDP in 2060 can be readily determined.

As the goal of this study is to capture the macroeconomic relationship between infrastructure investment and economic growth, different types of infrastructure and industries are not distinguished within the model. Similarly, all levels of government are consolidated into a single government entity which has average effective tax rates that accord with the total paid by employees and employers. The primary factor driving these simplifications is the lack of data related to the interaction between various infrastructure and industry types. While studies have examined some pairwise connections between specific industry types and specific infrastructure classes, in particular between the transportation industry and transport infrastructure (i.e. 21; 22), the general impacts are unknown. For example, the impact of water system infrastructure upon productive capacity of the transportation industry is unknown. However, data do exist when looking at infrastructure as a whole and its relationship with total production (13; 20; 5).
2.2 Data Quality and Assumptions

2.2.1 Data Quality

All of the data used in this study were obtained from the Statistics Canada CANSIM database. Appendix B provides a detailed list of the data used and the corresponding sources in the CANSIM database. Generally, data exist until 2009, however, due to data collection and processing, maintenance data are only current up to 2008. Furthermore, Statistics Canada had changed the reporting criteria for capital investment and maintenance in 1994. It was assumed that the ratio of maintenance to total investment would be consistent under either definition.

2.2.2 Assumptions

Many economic processes are difficult to model, and in the context of this study, would have little impact on the overall conclusions. For example, internal decisions within a corporation on whether to invest revenue in new capital or keep it as profit are outside the scope of this model. Therefore, historical behaviour is used to approximate these processes in the future and they are provided as exogenous inputs to the model. A total of six such exogenous inputs are used in this model. These include:

• Pre-tax profits for corporations
• Non-tax revenue for governments
• Government expenses not associated with wages or infrastructure
• Private capital depreciation rates
• Multi-factor productivity Consumption not associated with infrastructure or private investment

Appendix C.3 provides the supporting data for all of the assumptions used. In addition, it is assumed that a government will always attempt to eliminate any surplus or deficit solely through changes in tax rates. In practice, if a government were to generate a deficit due to increased infrastructure investment, it may also elect either to simply carry the deficit, or to reduce investment in other areas. However, in this analysis it was assumed that neither of these would occur.
3.0 Detailed Results Based on a Current Trend Policy and a Zero Marginal Return Policy

As stated in Section 1, the objective of this analysis is to personalize the risk of following different infrastructure policies to the various infrastructure stakeholders. While the complete range of potential policies is studied, there are two specific cases that are examined in more detail. The first case considers a scenario in which recently observed infrastructure investment policies are maintained in the future. In this situation, total infrastructure investment would be 3.1 per cent of GDP with maintenance investment averaging about 13 per cent. For reference, this will be referred to as the current trend policy. Under the reasonable assumption, based on the historical data, that current infrastructure investment levels are too low, the second policy of interest considers a scenario in which any further increase in infrastructure investment yields no more gains for the stakeholders. This infrastructure policy will be referred to as the zero marginal return policy.

3.1 The Zero Marginal Return Policy

The point at which marginal benefits are exhausted from infrastructure investment is referred to as the zero marginal return policy. The zero marginal return policy may depend on which stakeholder is asking the question. That is, the point at which marginal benefits are exhausted from infrastructure investment from a corporate point of view may differ from an employee’s perspective. Therefore, the zero marginal return policy was considered from three different points of view. These include:

- Societal point of view: maximum real GDP in 2060
- Employer point of view: maximum NPAT
- Employee point of view: maximum after-tax wages

A multi-dimensional maximization algorithm is used to determine the infrastructure policies that yield the largest real values for each of the quantities above.

Table 1 shows the three zero marginal return infrastructure policies obtained for each point of view. All three zero marginal return policies lie quite close to each other. This is a reassuring result since it means that by adopting the average policy, any individual stakeholder will still be quite close to their own maximum. It also implies that there should be very little friction between stakeholders when deciding on infrastructure policy.
At the zero marginal return policy, further increases in infrastructure investment yield no benefit. Figure 3 shows the relationship between average annual real rate of infrastructure investment growth compared to the average annual real rate of GDP growth by 2060. At the zero marginal return policy, any change in infrastructure investment—whether an increase or decrease—results in a lower rate of GDP growth. However, around this point, the decrease in GDP growth rate would be relatively small. In contrast, in the region around the current trend policy, the rate of GDP growth is very sensitive to changes in infrastructure investment. Even relatively small increases in infrastructure investment result in a significant change in the rate of GDP growth.

The increase from the current trend policy to the zero marginal return policy represents an increase in total infrastructure investment of 62 per cent. The net increase in maintenance is even greater at 179 per cent of current levels, since both the total infrastructure investment and the fraction of that devoted to maintenance have increased.

<table>
<thead>
<tr>
<th>Maximization Target</th>
<th>Total Infrastructure Spending (% of GDP)</th>
<th>Maintenance Fraction (% of total infrastructure spending)</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDP</td>
<td>5.4%</td>
<td>21.6%</td>
</tr>
<tr>
<td>After-tax wages</td>
<td>4.8%</td>
<td>23.7%</td>
</tr>
<tr>
<td>Net Profit After Tax</td>
<td>4.9%</td>
<td>22.0%</td>
</tr>
<tr>
<td>Average</td>
<td>5.0%</td>
<td>22.5%</td>
</tr>
</tbody>
</table>

Table 1: Infrastructure policies for maximum real GDP, net profit after tax, and after tax wages. The final row shows the average adopted for the remainder of the analysis.

Figure 3: The relationship between average annual real rate of change of infrastructure and the real rate of GDP growth. The triangle indicates the results if the current trend infrastructure policy is followed and the circle indicates the zero marginal return policy.

At the zero marginal return policy, further increases in infrastructure investment yield no benefit. Figure 3 shows the relationship between average annual real rate of infrastructure investment growth compared to the average annual real rate of GDP growth by 2060. At the zero marginal return policy, any change in infrastructure investment—whether an increase or decrease—results in a lower rate of GDP growth. However, around this point, the decrease in GDP growth rate would be relatively small. In contrast, in the region around the current trend policy, the rate of GDP growth is very sensitive to changes in infrastructure investment. Even relatively small increases in infrastructure investment result in a significant change in the rate of GDP growth.

The increase from the current trend policy to the zero marginal return policy represents an increase in total infrastructure investment of 62 per cent. The net increase in maintenance is even greater at 179 per cent of current levels, since both the total infrastructure investment and the fraction of that devoted to maintenance have increased.
The current infrastructure investment trend puts the Canadian economy at risk of not maximizing its future economic potential. The 3D illustration in Figure 4 shows the relationship between total public infrastructure investment (as a percentage of GDP), maintenance investment and the 2060 real value of GDP as a percent of current value. An adequately funded, stable, long-term infrastructure policy secures the foundation of efficient long-term economic production (the plateau). It is evident that the adoption of the recent trend of about 3.1 per cent of GDP for the next 50 years puts this foundation on a slippery slope, a place where small reductions in infrastructure investment or misallocations have a disproportionate impact upon the deterioration of economic growth. A direct comparison of annual GDP growths, shown in Figure 5, highlights the difference in real GDP growth under the two scenarios. While the difference in any given year is at most 1.3 per cent, the effects of compounding over 50 years results in a significant divergence in total real GDP by 2060.

Figure 4: The real GDP in 2060 as a percentage of real GDP today in terms of total infrastructure spending and the maintenance fraction. The white line indicates no change in GDP. The colour gradient highlights the percentage change in real GDP from today ranging from dark red where the real GDP has decreased, to green where the real GDP has approximately tripled.
While total real GDP provides a measure of the wealth of the economy as a whole, the real GDP per capita provides a better reflection of individual wealth. Figure 6 illustrates the divergence of per capita GDP under the two policy scenarios. Strikingly, the current policy trend appears to fall below what is expected from the historical trend. In contrast, the zero marginal return policy yields values that appear much more consistent with, and slightly above, historical trends. At risk is over 40 per cent of the potential real per capita GDP by 2060.
3.3 Fostering Better Real Wage Potential for Individuals

Employees and their employers have a stake in ensuring an adequately funded, stable, long-term infrastructure policy. If the potential risk of the underinvestment identified were to occur, it is estimated that employees are at risk of forgoing, on average, a 0.5 per cent per annum increase of their real after-tax income.

The full effect of this is best exemplified when looking at the implications over the working life of employees. For current employees born between 1985 and 1995, this means that their cumulative after-tax real wage is at risk of being between 25 per cent and 35 per cent less on average. For current employees born between 1965 and 1975, at risk is 16 per cent to 25 per cent of their cumulative after-tax real wage. When all future employees are taken into account, including those who immigrate and people born after 1995, maintaining the current trend policy over the next 50 years instead of the zero marginal return policy could mean a difference of 56 per cent in average real wages in 2060. Figure 7 shows the increasing benefit of the zero marginal return policy in terms of the average after-tax real wages.

Put into the perspective of a lump sum valued today a currently employed Canadian would be expected to pay the following for the identified long-term underinvestment in Canadian infrastructure:

- If born after 1995: greater than $51,000
- If born between 1985 and 1995: between $47,000 - $51,000
- If born between 1975 and 1985: between $40,000 - $47,000
- If born between 1965 and 1975: between $28,000 - $40,000
- If born before 1965: between $9,000 - $28,000
These results are in present value terms which reflect the current worth of future after-tax wage benefits. Future increases in after-tax wages are discounted at consumer borrowing rate, which was assumed to be 5.2% premium above the Federal Government borrowing rate, to reflect the time value of money. The future value of such benefits can then be easily calculated. For example, a fifteen year old in 2010 who later enters the workforce has an expected working life opportunity loss of $51,000. If invested at an interest rate of say 3.9% for 50 years (presumed age of retirement), the future value by 2060 would be $345,000.

In order to support the increased level of infrastructure investment, tax rates in the model do increase slightly. Figure 8 shows the tax rates under the current trend policy and the zero marginal return case. While tax rates do increase, they are generally below historical highs. However, even with higher tax burdens, there is a significant net benefit to employees. The present values of the difference in after-tax wages between the zero marginal return policy and the current trend policy exceeds the present value difference in taxes paid by 48 per cent. In other words, for every dollar increase in taxes paid, employees are better off by $1.48 in real after-tax wages. Towards the end of the simulation, tax rates start to decrease as the economy can now naturally support higher level of infrastructure investment.

Figure 8: Personal, corporate and consumption tax rates in the current trend policy and zero marginal return policy.
3.4 Increasing Net Profit After-Tax Return Potential For Business

Employers are not immune from the dangers of long-term underinvestment in infrastructure. If the potential risk of underinvestment were to occur, it is estimated that employers are at risk of forgoing, on average, a 0.68 per cent per annum increase of their NPAT. This impact accumulates over time as shown in Figure 9.

Over the next ten years, this would mean an average NPAT forgone by each employer of three per cent. Over the next 50 years, employer NPAT is at risk of being 20 per cent less on average. That is, the longer the exposure to underinvestment, the larger the impact on Canadian employers over time. This has the undesirable consequence of placing the costs of infrastructure underinvestment disproportionately upon those businesses that have long-term investment plans in Canada.

Figure 9: Average yearly net profit after-tax (NPAT) forgone if infrastructure policy were to follow the current trend rather than the zero marginal return policy.
3.5 Stable Infrastructure Investments are Preferable to Sporadic Spending

A key aspect of the previous analysis was that it assumed stable infrastructure investment. After the initial transition from the current infrastructure policy to the zero marginal return policy, the same percentage of GDP was allocated to infrastructure every year. Historically, such stable investment has not been observed. This raises the question of how unstable investment would affect returns on infrastructure spending.

To estimate the historical volatility in infrastructure investment, a smooth curve is fit to historical observations and the sum of two periodic triangular waves is fit to the fractional residuals. Figure 10 shows the results of the volatility estimate. This volatility factor is then applied to the future infrastructure policies in the analysis. In this way, the long term average infrastructure investment (as a percentage of GDP) is maintained and the volatility of the infrastructure investment is reflective of the trends seen historically. In order to isolate the effect of irregular investment from other dynamic effects in the model, future tax rates from the stable investment model are used. This assumption effectively means that governments in the unstable investment scenarios would divert funds from other programs at times when infrastructure investment is above average, and spend funds elsewhere when infrastructure investment is below average. The result is the same surplus and deficits as in the stable case.

Figure 10: A) The smooth base trend in the historical data. B) The residuals from the smooth trend and the results of fitting them.
The impact of unstable investment is most apparent when examining the marginal rates of return. Figure 11 shows how the average annual rate of GDP growth by 2060 is less for a given rate of change in infrastructure investment under irregular investment models than with stable investment. As the level of volatility increases, the reduction in the rate of growth of GDP continues to increase.

Figure 11: The impact of irregular funding on rate of GDP growth and infrastructure spending.
It’s also of note that, on the irregular investment curves, the policy parameters corresponding to the point of zero marginal return in the stable investment scenario move to the left along the curve. This means that even though the average fraction of GDP being spent on infrastructure over many years is the same, there is a second-order effect whereby the irregular investment reduces growth resulting in fewer available funds to support infrastructure. Therefore, if one wanted to achieve the same rate of GDP growth under an unstable investment scenario, the average fraction of GDP spent on infrastructure would have to be significantly larger than in the stable investment case, resulting in not only higher total investment, but also a greater average fraction of GDP devoted to infrastructure. Figure 12 highlights that for GDP growth rates greater than about 2.25 per cent, it is not possible to reach the same growth rate regardless of the level of infrastructure investment. Unstable or short-term infrastructure policies introduce additional frictions to the economic productive process. The result is that infrastructure investment does not translate as well into economic growth when it is volatile.

Figure 12: Real GDP in 2060 with unstable infrastructure funding at historic levels of volatility.
3.6 Sensitivity to the Elasticity of Public Infrastructure

The largest uncertainty when performing an analysis of the impact of infrastructure spending on the overall economy is the elasticity of production for public capital, $\beta_g$. The value of 0.1 used in the analysis is the best estimate from Macdonald (13), however this study also provides a possible range of 0.05 to 0.15. A lower value would mean that public infrastructure contributes less to economic production. In the limit of $\beta_g = 0$, public infrastructure does not contribute at all to production. In contrast, a larger value would increase the importance of public infrastructure. In order to check the sensitivity of the conclusions to the elasticity of production for public capital, the basic analysis described in Section 3.1 is repeated for $\beta_g = 0.05$ and $\beta_g = 0$, but with a focus upon GDP only. Table 2 shows the zero marginal return policies for GDP for each of the elasticities under consideration. The larger the elasticity, the greater the benefit of investing in infrastructure. However, even in the case with the lowest elasticity, the total infrastructure investment which yields the largest GDP in 2060 is still significantly above the current trend values. Figure 13 shows that in either case, the current trend policy lies significantly below zero marginal return policies. Therefore, while the exact numerical values of future GDP and wages may change under different values of elasticity of production for infrastructure, the general conclusion that the current policy trend puts significant economic growth at risk remains valid.

<table>
<thead>
<tr>
<th>Elasticity</th>
<th>Total Infrastructure Spending (% of GDP)</th>
<th>Maintenance Fraction (% of total infrastructure spending)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>4.7%</td>
<td>24.5%</td>
</tr>
<tr>
<td>0.1</td>
<td>5.4%</td>
<td>21.6%</td>
</tr>
<tr>
<td>0.15</td>
<td>6.5%</td>
<td>18.9%</td>
</tr>
</tbody>
</table>

Table 2: The zero marginal return infrastructure policies for GDP for the range of elasticities of production for infrastructure.

Figure 13: The relationship between average annual real rate of change of infrastructure and the real rate of GDP growth for elasticities of 0.05 (left) and 0.15 (right). The triangles indicate the results if the current trend infrastructure policy is followed and the circles indicate the zero marginal return policy.
4.0 Conclusions

4.1 Overview
Given the significant discrepancy between the current infrastructure investment trend and the trend required to exhaust the marginal economic production benefits, a macroeconomic case is made for the identification of risk of long-term public infrastructure underinvestment. When coupled with recent estimates of a significant infrastructure deficit in Canada, the identified risk of infrastructure underinvestment is persuasive and in need of attention.

Total public infrastructure investment over the last decade has averaged about 3.1 per cent of GDP (of which about 13 per cent is spent on repair and maintenance). The macroeconomic analysis conducted suggests that total public infrastructure investment could be increased up to five per cent of GDP (22.5 per cent of which would be for repair and maintenance) before exhausting the marginal economic production benefits. Furthermore, the analysis shows that the rate of return on infrastructure investment is increased when the volatility of infrastructure investment is decreased.

The identified risk of infrastructure underinvestment has implications for the future of the Canadian economy and its participants. It puts the Canadian economy at risk of not maximizing its future economic potential and has consequences for Canadian employees and employers. The consequences for employees are material and are disproportionately placed on the shoulders of younger labour-force participants. The consequences for employers are also material and are disproportionately placed on the shoulders of businesses that have long-term investment plans in Canada.

A key aspect of this analysis was the assumption of stable public infrastructure investment. Historically, such stable investment has not been observed. This raises the question of how unstable investment would affect returns on infrastructure spending. The impact of such unstable investment is most apparent when examining the marginal rates of return. Analysis conducted shows that the average annual rate of GDP growth by 2060 is less for a given rate of change in infrastructure investment under irregular investment models than with stable investment. As the level of volatility increases, the reduction in the rate of growth of GDP continues to increase. A second-order effect is also noted, whereby the irregular investment reduces growth, resulting in fewer available funds to support infrastructure. Therefore, if one wanted to achieve the same rate of GDP growth under an unstable investment scenario, the average fraction of GDP spent on infrastructure would have to be significantly larger than in the stable investment case resulting in not only higher total investment, but also a greater average fraction of GDP being devoted to infrastructure.

The macroeconomic analysis in this report considers only the relationship between infrastructure investment and key macroeconomic variables. It does not consider the details of infrastructure inventory currently on hand, or how such an inventory should
change over time. The analysis, therefore, only considers whether the current long-
term investment trend makes sense from a macroeconomic point of view.

Given the identified risk of infrastructure underinvestment, it is strongly recommended
that further analysis be conducted that couples national infrastructure inventory and
deficit estimates with macroeconomic policy analysis. The collaboration of these types
of datasets and analysis is necessary to ascertain an actionable strategy that can mitigate
the identified risk of infrastructure underinvestment in Canada.

4.2 Limitations
The conclusions of this study are heavily dependent on the assumption of a Cobb-
Douglas production function using a single infrastructure capital quantity. In reality,
there are many different types of infrastructure, each with a different impact on
production in the economy. For example, the construction of a new highway may have
a significantly different elasticity of production than a new arena. The relative amount
of different infrastructure types may change over time, resulting in a different elasticity
of production for infrastructure as a whole. The assumption of a constant elasticity of
production effectively assumes that future investments in infrastructure will reflect the
same productivity capacity as in the past.

The economic model considers Canada as an isolated economy. Imports and exports
are not considered. A larger increase in Canadian production would be possible if
Canada’s trade surplus was allowed to grow. However, the inverse may also occur if
increased imports were to lower domestic production requirements. Since international
trade balances will depend on the behaviour of the economies of countries outside
Canada, this is outside the scope of this project. It is also assumed that population
growth is not influenced by economics. In particular, immigration policies are not
influenced by changing labour market demands. In scenarios with higher growth,
the demand for employees may exceed local availability which acts to increase real
wages. Allowing immigration rates to increase in these situations would alter both
the demographic and economic outcomes. However, this type of decision is largely
political and modelling these scenarios is outside the scope of this study.

Finally, the impact of infrastructure on more intangible quantities such as quality
of life is not addressed in this study. For example, improvements in transportation
infrastructure may result in reduced commuting times, which may not affect companies
directly, but would have a bearing on the overall quality of life for the employee.
Similarly, community buildings such as arenas or community centres may not have a
significant long-term impact on production, but could lead to a higher quality of life.
Such benefits are not readily captured in a macroeconomic model.
4.3 Future Research

An important direction of future research involves breaking down infrastructure assets into various classes (such as transportation, water systems, and community infrastructure such as arenas). A model with finer asset divisions, in conjunction with a detailed inventory of the existing infrastructure inventory, could begin to provide recommendations on the type of infrastructure that is most beneficial to the economy.

Closely related to the research into various infrastructure asset categories would be research into different private industry areas. Different types of infrastructure may benefit certain types of industry more than others, allowing fine-tuned infrastructure policies to be developed. However, significant research into the effect of each type of infrastructure on each industry would be required to support such modelling efforts. In this regard, this report is intended as an initial step in a larger project that can be expanded upon in the future with finer infrastructure and industry categories. The econometric modelling approach has the advantage of being able to apply sensitivity analysis and using an expanded range of variables to investigate the dependence of economic growth upon unknown or poorly known quantities.
APPENDIX A. Bibliography


The data used in this study were obtained from the Statistics Canada Canadian Socio-Economic Information Management System (CANSIM). RiskAnalytica’s version of the database is current as of October 2008 and was supplemented with updates from the CANSIM website. Where the quantity is used directly in the mathematical model, the symbol used to represent it is also indicated.

### APPENDIX B. Data Sources

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>CANSIM Table</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>Demographics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Population</td>
<td>The population of Canada by age and sex from 1971 to 2007.</td>
<td>051-0001</td>
<td>$P_{ig}$</td>
</tr>
<tr>
<td>Births</td>
<td>The number of births in Canada by sex from 1971 to 2007</td>
<td>051-0013</td>
<td>$B_{ig}$</td>
</tr>
<tr>
<td>Deaths</td>
<td>Number of deaths in Canada by age and sex</td>
<td>051-0002</td>
<td>$D_{ig}$</td>
</tr>
<tr>
<td>Immigration</td>
<td>Immigration into Canada by age and sex</td>
<td>051-0012</td>
<td></td>
</tr>
<tr>
<td>Emigration</td>
<td>Emigration from Canada by age and sex</td>
<td>051-0012</td>
<td></td>
</tr>
<tr>
<td>Economics</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td>Annual gross domestic product from 1961 to 2009</td>
<td>380-0016</td>
<td>$Y$</td>
</tr>
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<td>Pre-tax Corporate Profits</td>
<td>Annual pre-tax profits from 1961 to 2009</td>
<td>380-0016</td>
<td>$\Phi_{Pre-tax}$</td>
</tr>
<tr>
<td>Total Wages</td>
<td>Annual total wages from 1961 to 2009</td>
<td>380-0016</td>
<td>$L$</td>
</tr>
<tr>
<td>Real Average Wages</td>
<td>Real average wages (in 2006 dollars) by age and sex from 1976 to 2006</td>
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<td>$W_{ig}$</td>
</tr>
<tr>
<td>Labour force statistics</td>
<td>Employed, unemployed and non-labour by age and sex from 1976 to 2007</td>
<td>282-0002</td>
<td>$E_{ig}, U_{ig}, N_{ig}$</td>
</tr>
<tr>
<td>Government Employees</td>
<td>Public sector employment by age and sex from 1976 to 2007</td>
<td>282-0008</td>
<td>$E_{ig}$</td>
</tr>
<tr>
<td>Capital</td>
<td>Public and private capital stocks</td>
<td>031-0002</td>
<td>$G, K$</td>
</tr>
<tr>
<td>Investment</td>
<td>Public and private capital investment</td>
<td>031-0002</td>
<td>$I_{g}, I_{f}$</td>
</tr>
<tr>
<td>Infrastructure Maintenance</td>
<td>Government investment on infrastructure capital, repair and maintenance from 1956 to 1994, and 1994 to 2009</td>
<td>029-0005, 029-0035</td>
<td>$M$</td>
</tr>
<tr>
<td>Government Accounts</td>
<td>Government expenses and revenue from 1981 to 2009</td>
<td>384-0004</td>
<td>$E, R$</td>
</tr>
<tr>
<td>Consumer price index</td>
<td>Consumer price index from 1914 to 2009</td>
<td>325-0021</td>
<td></td>
</tr>
</tbody>
</table>

2 http://cansim2.statcan.ca/
C.1 Demographic Model

The demographic model incorporates birth, deaths, migration, and aging to model the future population of Canada.

C.1.1 Birth

The birth rate is assumed to be proportional to the number of females between the ages of 15 and 45,

\[ \dot{B}^g = \beta^g \sum_{a=15}^{45} P^g,\text{female} \]

where the proportionality constant is assumed to follow an decreasing trend estimated from the historical values.

C.1.2 Death

Death is handled in a manner similar to birth. For each age-group, there is a rate coefficient characterizing their likelihood of death.

\[ \dot{\mu}^g = \mu^g p^g \]

Again, the values of \( \mu^g \) are assumed to follow a decreasing trend based on historical values.

C.1.3 Migration

Immigration and emigration have a significant impact on the growth of the Canadian population. Unlike birth and death, where trends of decreasing births and deaths are apparent, no such trend exists in the immigration and emigration rates. Therefore, immigration and emigration rates per capita, \( \alpha_{\text{imm}} \) and \( \alpha_{\text{em}} \) respectively, are assumed to be a constant and based on the historical average. Therefore,

\[ \dot{p}^g_{\text{Migration}} = \alpha_{\text{imm}} p^g - \alpha_{\text{em}} p^g \]

C.1.4 Aging

The last component in the demographic model is the aging of the population. The aging model adopted is

\[ \dot{p}^g = \frac{p^{(a-1),g}}{\Delta_{a-1}} - \frac{p^g}{\Delta_a} \]

where the first term describes the people aging from age-group \( a-1 \) into age-group \( a \), and the second term describes the people aging out of age-group \( a \) into \( a+1 \). The width of age-group \( a \) is \( \Delta_a \).
C.2 Economic Model

Private production is the primary driver of the economy. A simple, but surprising robust, method to model private output is to use a Cobb-Douglas model (19) which relates the amount of capital available in different classes to the rate of production. It states that:

\[ Y = \alpha \prod_i K_i^{\beta_i} \]

where \( Y \) is the rate of production, \( K_i \) is the value of capital class \( i \), \( \beta_i \) is the elasticity of capital class \( i \), and \( \alpha \) is the multi-factor productivity term. The multi-factor productivity term accounts for the increasing productivity per capital unit due to non-tangible effects such as technological advancements. Two classes of capital are always considered: private capital, \( K \), and labour capital, \( L \). To account for the relative productivity of employees of varying experience, labour capital is defined using average real wages and employment numbers such that:

\[ L = \sum_{ag} E_{ag} w_{ag} \]

where \( E_{ag} \) are the number of people employed in age group \( a \) of gender \( g \), and \( w_{ag} \) is the average real wage paid to a person in that group. In order to capture the effects of infrastructure upon production, public capital, \( G \), is a third class of capital considered in the economic model. Therefore, the production function used is

\[ Y = \alpha K^{\beta_k} L^{\beta_L} G^{\beta_G} \]

An extensive analysis of the role of public capital in Canadian production (13) found reasonable elasticities of capital to be:

\[ \beta_k = 0.31 \]
\[ \beta_L = 0.69 \]
\[ \beta_G = 0.1 \]

Using these elasticities, along with the historical capital values and GDP, the historical multifactor productivity factor can be calculated.
C.2.1 Public and Private Capital

The dynamics of public and private capital are based upon modelling done by Dioikitopoulos and Kalyvitis (20; 5). To determine how production changes over time, the time derivatives of the capital classes are required. For public and private capital, new investment will act to increase capital values, while depreciation will act to decrease capital. In particular:

\[
\dot{K} = I_K - \delta_K K
\]
\[
\dot{G} = I_G - \delta_G G
\]

where \(I_K\) and \(I_G\) are new investments in private and public capital, and \(\delta_K\) and \(\delta_G\) are the depreciation rates of private and public capital. The historical trend of the private depreciation rate is extrapolated into the future.

The depreciation rate of public capital has an additional influence which affects its value. As production increases, there is increased stress placed upon the public infrastructure. For example, as factories increase output, there is an increased burden on the transportation infrastructure causing it to depreciate at a faster rate. To compensate for this additional wear-and-tear, governments can perform maintenance on the infrastructure which acts to reduce its depreciation rate. To account for these effects, the depreciation rate of public capital is taken to be a function of both maintenance and production (20). Therefore, the equations governing the rate of change in public capital are:

\[
\dot{G} = I_G - \delta_G \left( \frac{M}{Y} \right) G
\]
\[
\delta_G(t) = \left( \delta_{G_{\text{max}}} - \delta_{G_{\text{min}}} \right) e^{-\lambda t} + \delta_{G_{\text{min}}}
\]

The parameters \(\delta_{G_{\text{max}}}, \delta_{G_{\text{min}}},\) and \(\lambda\) can be estimated using a least-squares fit from historical data. Section C.3 presents the results of the fits for \(\delta_K\) and \(\delta_G\).

While government revenue is required to fund infrastructure projects, the fraction of production devoted to infrastructure, \(\tau\), is a primary policy parameter.

\[
\tau = \frac{M + I_G}{Y}
\]

The relative split of infrastructure between maintenance and new investment defined as:

\[
\mu = \frac{M}{M + I_G}
\]

is the second infrastructure policy parameter.
The total amount of private investment can be determined through the resource constraint equation

\[ Y = C + I_k + I_G + M \]

When consumption is defined according to this decomposition of production, the historical ratio:

\[ \sigma = \frac{C}{Y} \]

is relatively constant (see section C.3.1). Solving for the amount of private investment yields:

\[ I_k = Y(1-\tau-\sigma) \]

Unfortunately, as the total private investment must be positive, this form does not allow \( \tau + \sigma > 1 \). However, differentiating yields:

\[ \dot{I}_k = \dot{Y}(1-\tau-\sigma) - Y(\dot{\tau}+\dot{\sigma}) \]

which has no constraints and \( I_k \) becomes an integrated quantity.

### C.2.2 Target Production Growth

The rate of growth of production is closely tied to the rate of growth in infrastructure. Theoretical models tested on Canadian infrastructure (5) data have yielded a strong correlation between the rate of change of production and the rate of change of infrastructure. It was found that increases in private investment lag behind the increase in infrastructure investment indicating that it was the infrastructure growth supporting the production increases which in turn led to increased private investment rather than the other way around. Therefore, given the rate of change of infrastructure capital, \( \dot{G} \), one can calculate what the target change in production should be:

\[ \frac{\dot{Y}_{\text{Target}}}{Y} = \frac{\dot{G}}{G} \]

However, in order to reach this target production, other components of the economy must be able to respond accordingly.
C.2.3 Employment and Wages

Given the target rate of production, differentiating the production function allows one to determine the target rate of change of labour capital given that the private and public capital derivatives are known. Differentiating and solving yields:

\[ i_{\text{Target}} = \frac{L}{\beta} \left( \frac{Y_{\text{Target}}}{Y} \frac{\dot{a}}{a} - \beta_k \frac{\dot{K}}{K} - \beta_g \frac{\dot{G}}{G} \right) \]

However, as the change in labour capital is constrained by demographics, it may not be possible to achieve the target labour capital growth. The rate of change in labour capital, \( L \), is a combination of changing employment rates and average wages.

\[ L = \sum_{ag} \left( \dot{E}^{ag} w^{ag} + E^{ag} \dot{w}^{ag} \right) \]

There are several processes that affect changes in employment for each age/sex group. These include aging into and out of the age-group, deaths, retirement, new hiring, and other processes such as migration. Similar processes affect the unemployed, \( U^{ag} \), and the non-labour, \( N^{ag} \), populations. Therefore:

\[
\begin{align*}
\dot{E}^{ag} &= A^{ag}(E) - \dot{D}_E^{ag} - \dot{R}^{ag} + \dot{H}^{ag} + \Delta_{E}^{ag} \\
\dot{U}^{ag} &= A^{ag}(U) - \dot{D}_U^{ag} - \dot{H}^{ag} + \dot{S}^{ag} + \Delta_{U}^{ag} \\
\dot{N}^{ag} &= A^{ag}(N) - \dot{D}_N^{ag} - \dot{S}^{ag} + \dot{R}^{ag} + \Delta_{N}^{ag}
\end{align*}
\]

where \( A^{ag}(X) \) is the aging operator for age-group \( a \) on population \( X \), \( \dot{D}_X^{ag} \) is the rate of deaths among the population \( X^{ag} \), \( \dot{R}^{ag} \) is the retirement rate, \( \dot{H}^{ag} \) is the hiring rate, \( \dot{S}^{ag} \) is the rate people move from non-labour to unemployed and \( \Delta_X^{ag} \) is the rate of change due to other processes. If one assumes that deaths among each employment group occur at the same rate as the population overall, then:

\[
\begin{align*}
\dot{D}_E^{ag} &= \frac{E^{ag}}{P^{ag}} \dot{D}^{ag} \\
\dot{D}_U^{ag} &= \frac{U^{ag}}{P^{ag}} \dot{D}^{ag} \\
\dot{D}_N^{ag} &= \frac{N^{ag}}{P^{ag}} \dot{D}^{ag}
\end{align*}
\]
Similarly, if one assumes that the changes due to other processes occur equally across employment groups:

\[
\Delta_{E}^{\text{eq}} = \frac{E_{t}}{P_{t}} \Delta_{E}^{\text{eq}} \\
\Delta_{U}^{\text{eq}} = \frac{U_{t}}{P_{t}} \Delta_{U}^{\text{eq}} \\
\Delta_{N}^{\text{eq}} = \frac{N_{t}}{P_{t}} \Delta_{N}^{\text{eq}}
\]

The total change due to other process, \( \Delta_{\text{eq}} \), can be determined by noting that:

\[
\dot{P}_{t}^{\text{eq}} = \dot{E}_{t}^{\text{eq}} + \dot{U}_{t}^{\text{eq}} + \dot{N}_{t}^{\text{eq}}
\]

so:

\[
\Delta_{t}^{\text{eq}} = \dot{P}_{t}^{\text{eq}} - A_{t}^{\text{eq}} (P) + \dot{D}_{t}^{\text{eq}}
\]

which is known from the demographic model. Retirement rates are assumed to be a constant fraction of the employed which can be modelled as:

\[
\hat{R}^{\text{eq}} = \gamma_{t}^{\text{eq}} E_{t}^{\text{eq}}
\]

where \( \gamma_{t}^{\text{eq}} \) can be estimated from historical data. Hiring rates, \( \hat{H}_{t}^{\text{eq}} \), are taken to be a constant fraction of the non-employed population such that:

\[
\hat{H}_{t}^{\text{eq}} = \alpha_{t}^{\text{eq}} \beta_{t}^{\text{eq}} (U_{t}^{\text{eq}} + N_{t}^{\text{eq}})
\]

where \( \alpha_{t}^{\text{eq}} \) is the overall demand factor such that \( \hat{L}_{\text{target}} \) can be met and \( \beta_{t}^{\text{eq}} \) is the hiring bias. As with \( \gamma_{t}^{\text{eq}} \), the value of \( \beta_{t}^{\text{eq}} \) can be estimated from historical employment data. Finally, \( \hat{S}_{t}^{\text{eq}} \) is chosen to maintain a constant unemployment rate, \( \mu_{t}^{\text{eq}} \), where:

\[
\mu_{t}^{\text{eq}} = \frac{U_{t}^{\text{eq}}}{E_{t}^{\text{eq}} + U_{t}^{\text{eq}}}
\]

Differentiating yields:

\[
\dot{U}_{t}^{\text{eq}} = \frac{\mu_{t}^{\text{eq}}}{1 - \mu_{t}^{\text{eq}}} \dot{E}_{t}^{\text{eq}}
\]
A final constraint placed on the hiring rate is that the net rate of change in the labour force rate does not exceed the maximum rate of change seen historically. The labour force rate is defined as:

\[ \nu^{ag} = \frac{E^{ag} + U^{ag}}{P^{ag}} \]

Therefore:

\[ \dot{\nu}^{ag} = \frac{\dot{E}^{ag} + \dot{U}^{ag}}{P^{ag}} - \nu^{ag} \frac{\dot{P}^{ag}}{P^{ag}} \leq \dot{\nu}_{\text{max}}^{ag} \]

Under the assumption of a constant unemployment rate, this yields:

\[ \dot{E}^{ag} \leq (1 - \mu^{ag}) \left( \nu^{ag}_{\text{max}} P^{ag} + \nu^{ag} \dot{P}^{ag} \right) \]

If the rate of change in labour capital is still less than the target rate of change, real wages can be adjusted in an attempt to make up the difference. The change in real wages is assumed to be proportional to the current wages:

\[ \dot{w}^{ag} \propto w^{ag} \]

Therefore:

\[ w^{ag} = \left( \frac{\dot{I}_{\text{target}}}{L} - \frac{1}{L} \sum_{ag} (\dot{E}^{ag} w^{ag}) \right) w^{ag} \]

Again, the rate of change in real wages is constrained by the rates of change seen historically.

C.2.4 Corporate Profits

Pre-tax corporate profits, \( \Phi_{\text{post-ax}} \), are assumed to be a multiple of total production which are then taxed at a rate of \( \tau_{\text{Corp}} \). Therefore:

\[ \Phi_{\text{post-ax}} = (1 - \tau_{\text{Corp}}) \Phi_{\text{pre-ax}} \]

\[ \Phi_{\text{pre-ax}} = f_{\text{pre-ax}} Y \]

Section C.3.2 shows the calibration functions used for \( f_{\text{pre-ax}} \).
C.2.5 Government

Governments must attempt to balance their revenues with their expenses. Revenue primarily arises through taxation of personal income, corporate profits, and general production. However, revenue can also come from other sources such as investment income. Average tax rates are calculated based on total wages, total pre-tax corporate profits, and total production. Therefore, the rate at which the government collects revenue is:

\[ R = L^\text{Income} + \Phi^\text{Pre-tax Corp} + Y^\text{Production} + R^\text{Other} \]

Based on historical data shown in section C.3, it is assumed that \( R^\text{Other} \) is a simple fraction of the total revenue:

\[ R^\text{Other} = f^\text{NotTax} \cdot R \]

Therefore, the total government revenue rate is:

\[ R = \frac{1}{1 - f^\text{NotTax}} \left( L^\text{Income} + \Phi^\text{Pre-tax Corp} + Y^\text{Production} \right) \]

In the context of this project, government expenses consist of wages to government employees, funding of infrastructure, servicing of accumulated debt, \( D \), and other expenses. Thus:

\[ E = W_G + I_G + M + iD + E^\text{Other} \]

It is assumed that other expenses are proportional to the total government wages so that:

\[ E^\text{Other} = \zeta W_G \]

and \( \zeta \) is estimated from historical data. The net rate at which the government accumulates a surplus is then:

\[ S = R - E \]
In an attempt to balance the rate of revenue, with the rate of expenses, the government must adjust tax rates. It is assumed that tax rates will be adjusted according to the size of the surplus or deficit relative to total revenue:

\[ t^k = -\alpha_t \frac{S}{R} \]

where \( \alpha_t \) is the tax response rate parameter.

### C.3 Economic Calibration Functions

Calibration functions account for unknown or un-modelled economic processes. For example, this study is not concerned with the manner in which corporations generate pre-tax profits. Therefore, the historical ratio between pre-tax profits and total production is used as a heuristic to account for this process. If there are consistent historical trends in the data, the value can be extrapolated into the future. As the last historical data value may not align with the long-term trend, the difference is smoothly absorbed through an exponential decay. If \( f_H(t) \) is the set of historical values which exist up to time \( t_H \) and the identified trend is \( \hat{f}(t) \), then the calibration function is:

\[
f(t) = \begin{cases} 
  f_H(t) & t < t_H \\
  \hat{f}(t) + (f_H(t_H) - \hat{f}(t_H))e^{-\lambda(t-t_H)} & t > t_H 
\end{cases}
\]

### C.3.1 Consumption

The fraction of production consumed by personal, business, and government consumption is the consumption ratio. In the context of this model, it is all goods and services produced excluding government investment on infrastructure and new private capital investments:

\[
f_{\text{Consumption}} = \sigma = \frac{C}{Y}
\]
Figure 14 show the historical values and the trend extrapolated into the future.

C.3.2 Corporate Pre-Tax Profits

Pre-tax profits are assumed to be a fraction of total production:

\[ f_{\text{Pre-tax}} = \frac{\phi_{\text{Pre-tax}}}{Y} \]

Figure 15 shows that historically this ratio has ranged from a low of just under 6% to a high of over 14% with no noticeable long-term trend. Therefore, the average of the historical values is assumed to be the long-term trend.
C.3.3 Government Factors

The government calibration factors consist of the non-tax revenue factor, and the “other expense” factor. Figure 16 and Figure 17 show the historical values and the extrapolation to the historical average for each factor respectively.

Figure 16: The fraction of government revenue from non-tax sources

Figure 17: The government expense factors defined as unaccounted for expenses as a multiple of government wages.
C.3.4 Depreciation Factors

Private Capital Depreciation

The rate of depreciation of private capital is estimated from the trend in the historical depreciation rates. As illustrated in Figure 18, there has been a consistent upwards trend in the depreciation rate. Therefore, this general behaviour is extrapolated linearly into the future.

![Private Depreciation Rate](image)

Figure 18: Depreciation rate of private capital.

Public Capital Depreciation

Figure 19 illustrates the dependence of the depreciation rate of public capital upon infrastructure maintenance investment and production. As total maintenance investment is \( M = \mu \tau Y \), one sees that:

\[
\delta_c \left( \frac{M}{Y} \right) = \delta_c (\mu \tau)
\]

As the fraction of production spend on infrastructure maintenance decreases \((\mu \tau \to 0)\), the depreciation rate of the infrastructure increases. Conversely, if the maintenance fraction is increased, the depreciation rate will fall, but it cannot be decreased below about 4% per year. Therefore, increasing levels of maintenance will have diminishing returns.
Figure 19: Depreciation rate of public capital as a function of maintenance spending and production.

**Multifactor Productivity**

The multifactor productivity has a direct impact upon future production in the model. The form adopted here extends the historical trend, while conservatively capping the value at 1.05.

Figure 20: Multifactor productivity factor
RCCAO members include: Carpenters’ Union • Greater Toronto Sewer and Watermain Contractors Association • Heavy Construction Association of Toronto • International Union of Operating Engineers, Local 793 • International Union of Painters and Allied Trades, District Council 46 • Joint Residential Construction Council • LIUNA Local 183 • Residential Carpentry Contractors Association • Toronto and Area Road Builders Association

The Residential and Civil Construction Alliance of Ontario (RCCAO) is an alliance composed of management and labour groups that represents a wide spectrum of the Ontario construction industry. The RCCAO’s goal is to work in cooperation with governments and related stakeholders to offer realistic solutions to a variety of challenges facing the construction industry. For more information on the RCCAO or to view copies of other studies and submissions, please visit the RCCAO website at www.rccao.com.